

Deep Reinforcement Learning

Deep Deterministic Policy Gradient

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Deterministic Policy Gradient Algorithms

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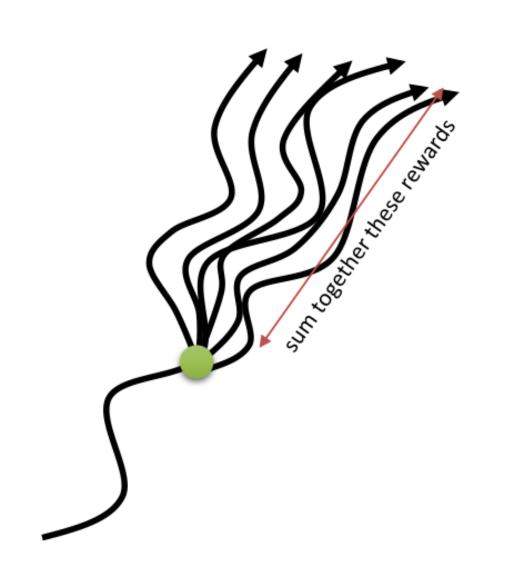
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Problems with stochastic policy gradient methods

 Actor-critic methods are strictly on-policy: the transitions used to train the critic must be generated by the current version of the actor.

$$egin{aligned}
abla_{ heta} \mathcal{J}(heta) &= \mathbb{E}_{s_t \sim
ho_{ heta}, a_t \sim \pi_{ heta}} [
abla_{ heta} \log \pi_{ heta}(s_t, a_t) \left(R_t - V_{arphi}(s_t)
ight)] \ \mathcal{L}(arphi) &= \mathbb{E}_{s_t \sim
ho_{ heta}, a_t \sim \pi_{ heta}} [\left(R_t - V_{arphi}(s_t)
ight)^2] \end{aligned}$$

- Past transitions cannot be reused to train the actor (no replay memory).
- Domain knowledge cannot be used to guide the exploration.
- The learned policy $\pi_{\theta}(s,a)$ is **stochastic**. This generates a lot of **variance** in the obtained returns, therefore in the gradients.
- This can greatly impair learning (bad convergence) and slow it down (sample complexity).
- We would not have this problem if the policy was deterministic as in off-policy methods.



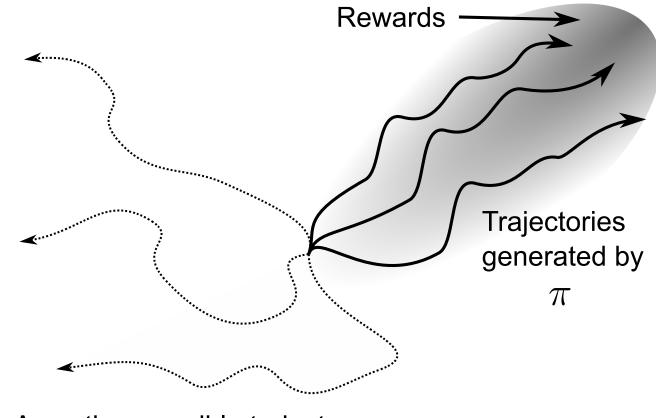
The objective function that we tried to maximize until now is:

$$\mathcal{J}(heta) = \mathbb{E}_{ au \sim
ho_ heta}[R(au)]$$

i.e. we want the returns of all trajectories generated by the **stochastic policy** π_{θ} to be maximal.

- It is equivalent to say that we want the value of **all** states visited by the policy π_{θ} to be maximal:
 - a policy π is better than another policy π' if its expected return is greater or equal than that of π' for all states s.

$$\pi > \pi' \Leftrightarrow V^\pi(s) > V^{\pi'}(s) \quad orall s \in \mathcal{S}$$



Any other possible trajectory

• The objective function can be rewritten as:

$$\mathcal{J}'(heta) = \mathbb{E}_{s \sim
ho_{ heta}}[V^{\pi_{ heta}}(s)]$$

where ρ_{θ} is now the **state visitation distribution**, i.e. how often a state will be visited by the policy π_{θ} .

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• The two objective functions:

$$\mathcal{J}(heta) = \mathbb{E}_{ au\sim
ho_ heta}[R(au)]$$

and:

$$\mathcal{J}'(heta) = \mathbb{E}_{s \sim
ho_{ heta}}[V^{\pi_{ heta}}(s)]$$

are not the same: ${\mathcal J}$ has different values than ${\mathcal J}'$.

• However, they have a maximum for the same **optimal policy** π^* and their gradient is the same:

$$abla_{ heta}\,\mathcal{J}(heta) =
abla_{ heta}\,\mathcal{J}'(heta)$$

- If a change in the policy π_{θ} increases the return of all trajectories, it also increases the value of the visited states.
- Take-home message: their **policy gradient** is the same, we have the right to re-define the problem like this.

$$g =
abla_{ heta} \, \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_{ heta}} [
abla_{ heta} \, V^{\pi_{ heta}}(s)]$$

When introducing Q-values, we obtain the following policy gradient:

$$g =
abla_{ heta} \, \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_{ heta}} [
abla_{ heta} \, V^{\pi_{ heta}}(s)] = \mathbb{E}_{s \sim
ho_{ heta}} [\sum_{a}
abla_{ heta} \, \pi_{ heta}(s,a) \, Q^{\pi_{ heta}}(s,a)]$$

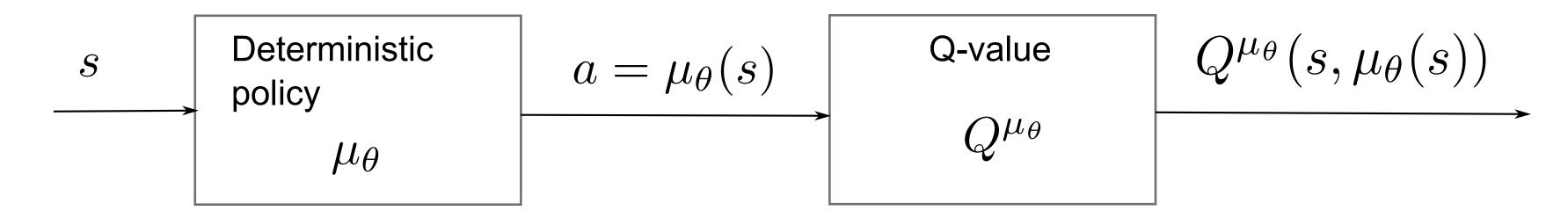
- This formulation necessitates to integrate overall possible actions.
 - Not possible with continuous action spaces.
 - The stochastic policy adds a lot of variance.
- But let's suppose that the policy is **deterministic**, i.e. it takes a single action in state s.
- We can note this deterministic policy $\mu_{ heta}(s)$, with:

$$egin{array}{ll} \mu_{ heta}: \; \mathcal{S}
ightarrow \mathcal{A} \ & s \;
ightarrow \mu_{ heta}(s) \end{array}$$

The policy gradient becomes:

$$g =
abla_{ heta} \, \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_{ heta}} [
abla_{ heta} \, Q^{\mu_{ heta}}(s, \mu_{ heta}(s))].$$

=



• The deterministic policy gradient is:

$$g =
abla_{ heta} \, \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_{ heta}} [
abla_{ heta} \, Q^{\pi_{ heta}}(s, \mu_{ heta}(s))]$$

• We can now use the chain rule to decompose the gradient of $Q^{\mu_{ heta}}(s,\mu_{ heta}(s))$:

$$abla_{ heta}\,Q^{\mu_{ heta}}(s,\mu_{ heta}(s)) =
abla_{a}\,Q^{\mu_{ heta}}(s,a)|_{a=\mu_{ heta}(s)} imes
abla_{ heta}\,\mu_{ heta}(s)$$

- $abla_a Q^{\mu_ heta}(s,a)|_{a=\mu_ heta(s)}$ means that we differentiate $Q^{\mu_ heta}$ w.r.t. a, and evaluate it in $\mu_ heta(s)$.
 - a is a variable, but $\mu_{ heta}(s)$ is a deterministic value (constant).
- $abla_{ heta} \mu_{ heta}(s)$ tells how the output of the policy network varies with the parameters of NN:
 - Automatic differentiation frameworks such as tensorflow can tell you that.

For any MDP, the **deterministic policy gradient** is:

$$abla_{ heta}\,\mathcal{J}(heta) = \mathbb{E}_{s\sim
ho_{ heta}}[
abla_{a}\,Q^{\mu_{ heta}}(s,a)|_{a=\mu_{ heta}(s)} imes
abla_{ heta}\,\mu_{ heta}(s)]$$

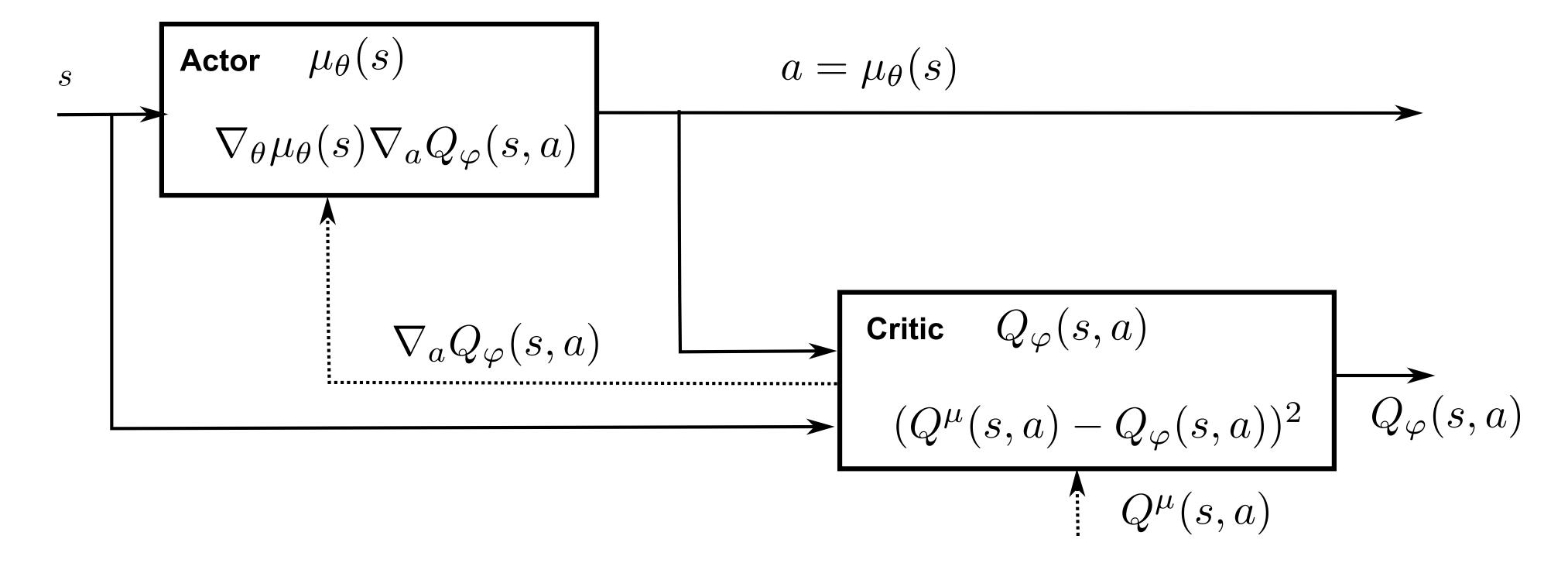
Deterministic policy gradient theorem with function approximation

- ullet As always, you do not know the true Q-value $Q^{\mu_ heta}(s,a)$, because you search for the policy $\mu_ heta.$
- (Silver et al, 2014) showed that you can safely (without introducing any bias) replace the true Q-value with an estimate $Q_{\varphi}(s,a)$, as long as the estimate minimizes the mse with the TD target:

$$egin{aligned} Q_{arphi}(s,a) &pprox Q^{\mu_{ heta}}(s,a) \ & \mathcal{L}(arphi) = \mathbb{E}_{s\sim
ho_{ heta}}[(r(s,\mu_{ heta}(s)) + \gamma\,Q_{arphi}(s',\mu_{ heta}(s')) - Q_{arphi}(s,\mu_{ heta}(s)))^2] \end{aligned}$$

- We come back to an actor-critic architecture:
 - The **deterministic actor** $\mu_{\theta}(s)$ selects a single action in state s.
 - The **critic** $Q_{\varphi}(s,a)$ estimates the value of that action.

Deterministic Policy Gradient as an actor-critic architecture



Training the actor:

$$abla_{ heta} \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_{ heta}} [
abla_{ heta} \, \mu_{ heta}(s) imes
abla_{a} Q_{arphi}(s,a)|_{a=\mu_{ heta}(s)}]$$

Training the critic:

$$\mathcal{L}(arphi) = \mathbb{E}_{s \sim
ho_{ heta}}[(r(s, \mu_{ heta}(s)) + \gamma \, Q_{arphi}(s', \mu_{ heta}(s')) - Q_{arphi}(s, \mu_{ heta}(s)))^2]$$

_

DPG is off-policy

• If you act off-policy, i.e. you visit the states s using a **behavior policy** b, you would theoretically need to correct the policy gradient with **importance sampling**:

$$abla_{ heta} \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_b}[\sum_a rac{\pi_{ heta}(s,a)}{b(s,a)} \,
abla_{ heta} \, \mu_{ heta}(s) imes
abla_a Q_{arphi}(s,a)|_{a=\mu_{ heta}(s)}]$$

- But your policy is now **deterministic**: the actor only takes the action $a=\mu_{\theta}(s)$ with probability 1, not $\pi(s,a)$.
- The **importance weight** is 1 for that action, 0 for the other. You can safely sample states from a behavior policy, it won't affect the deterministic policy gradient:

$$abla_{ heta} \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_b} [
abla_{ heta} \, \mu_{ heta}(s) imes
abla_a Q_{arphi}(s,a)|_{a=\mu_{ heta}(s)}]$$

- The critic uses Q-learning, so it is also off-policy.
- DPG is an off-policy actor-critic architecture!

2 - DDPG: Deep Deterministic Policy Gradient

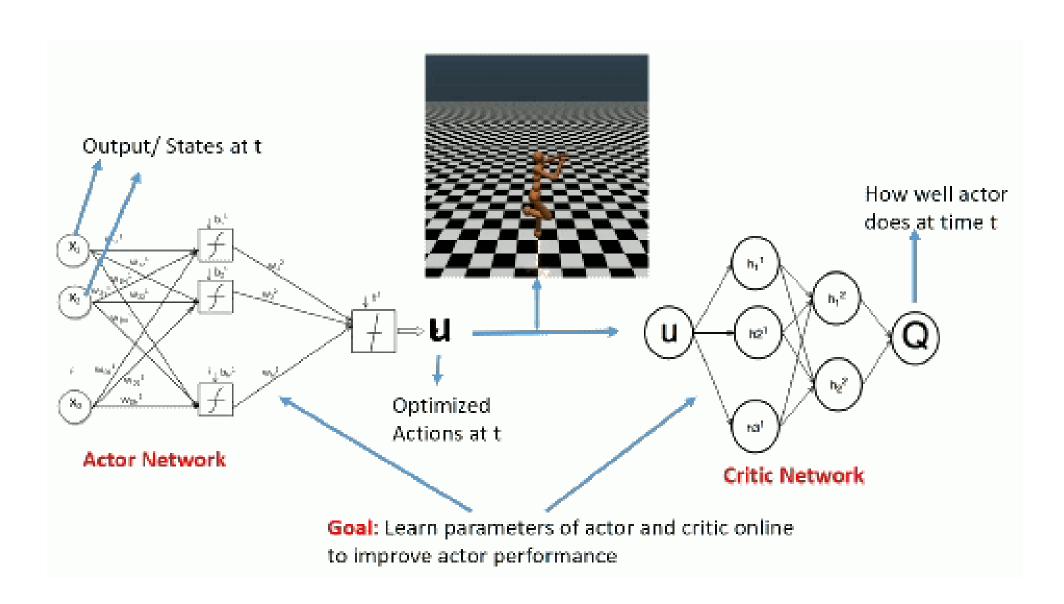
Published as a conference paper at ICLR 2016

CONTINUOUS CONTROL WITH DEEP REINFORCEMENT LEARNING

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DDPG: Deep Deterministic Policy Gradient

- As the name indicates, DDPG is the deep variant of DPG for continuous control.
- It uses the DQN tricks to stabilize learning with deep networks:



- As DPG is off-policy, an experience replay memory can be used to sample experiences.
- The actor μ_{θ} learns using sampled transitions with DPG.
- The **critic** Q_{φ} uses Q-learning on sampled transitions: **target networks** can be used to cope with the non-stationarity of the Bellman targets.

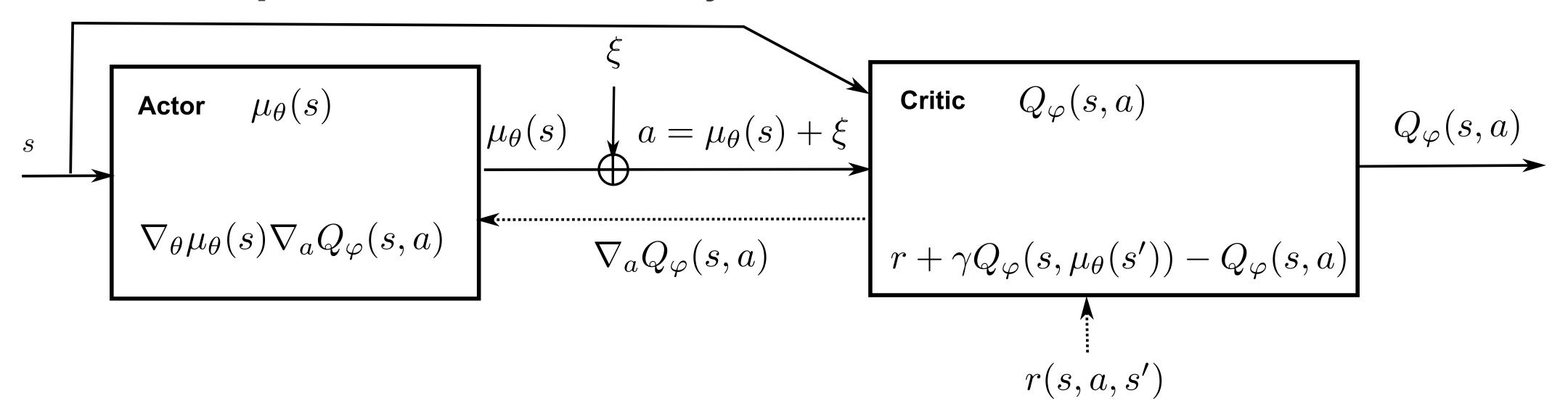
Source: https://github.com/stevenpjg/ddpg-aigym/blob/master/README.md

 Contrary to DQN, the target networks are not updated every once in a while, but slowly integrate the trained networks after each update (moving average of the weights):

$$\theta' \leftarrow \tau\theta + (1-\tau)\theta'$$

$$\varphi' \leftarrow \tau \varphi + (1 - \tau) \varphi'$$

DDPG: Deep Deterministic Policy Gradient



- A deterministic actor is good for learning (less variance), but not for exploring.
- We cannot use ϵ -greedy or softmax, as the actor outputs directly the policy, not Q-values.
- For continuous actions, an **exploratory noise** can be added to the deterministic action:

$$a_t = \mu_ heta(s_t) + \xi_t$$

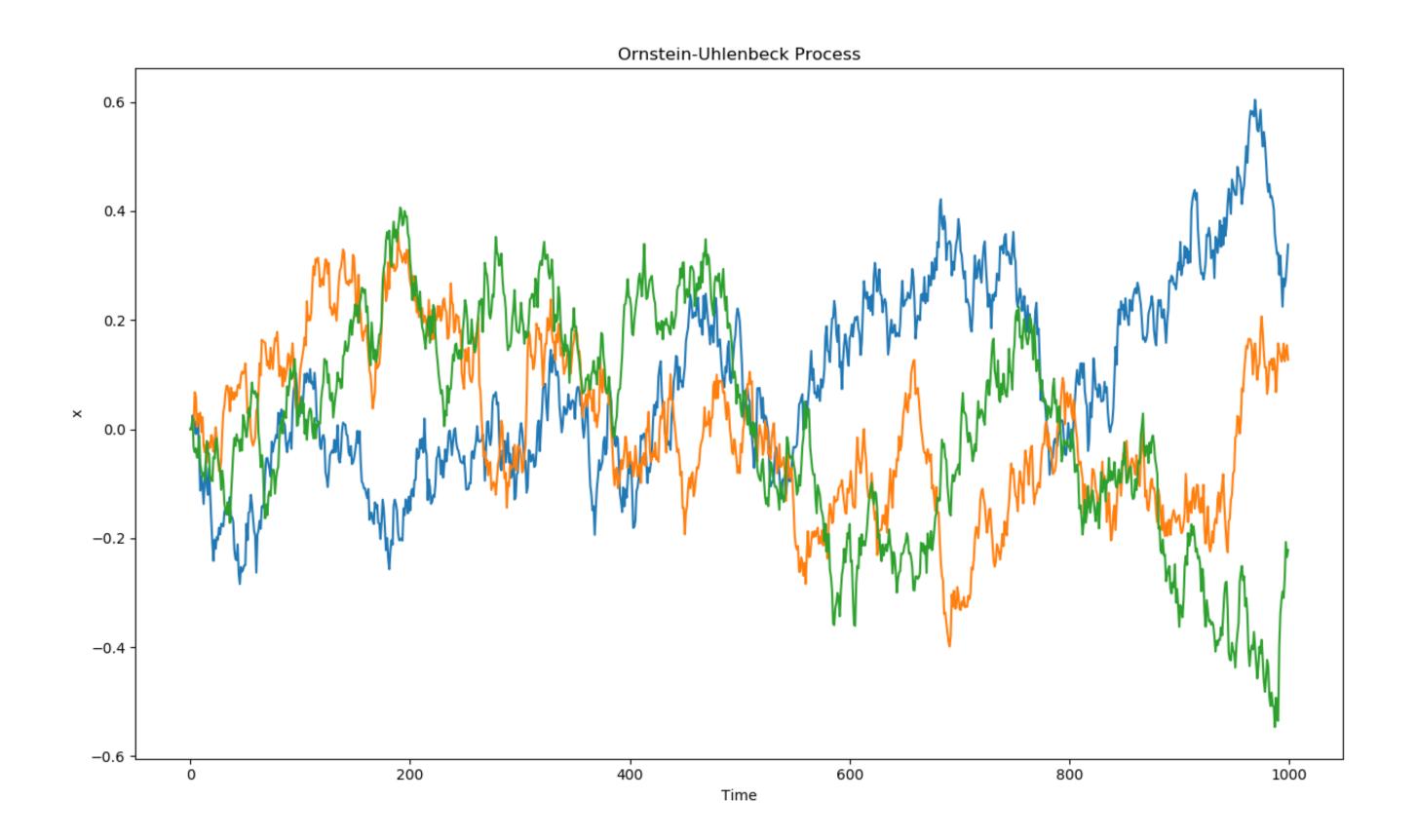
• Ex: if the actor wants to move the joint of a robot by 2^o , it will actually be moved from 2.1^o or 1.9^o .

Ornstein-Uhlenbeck stochastic process

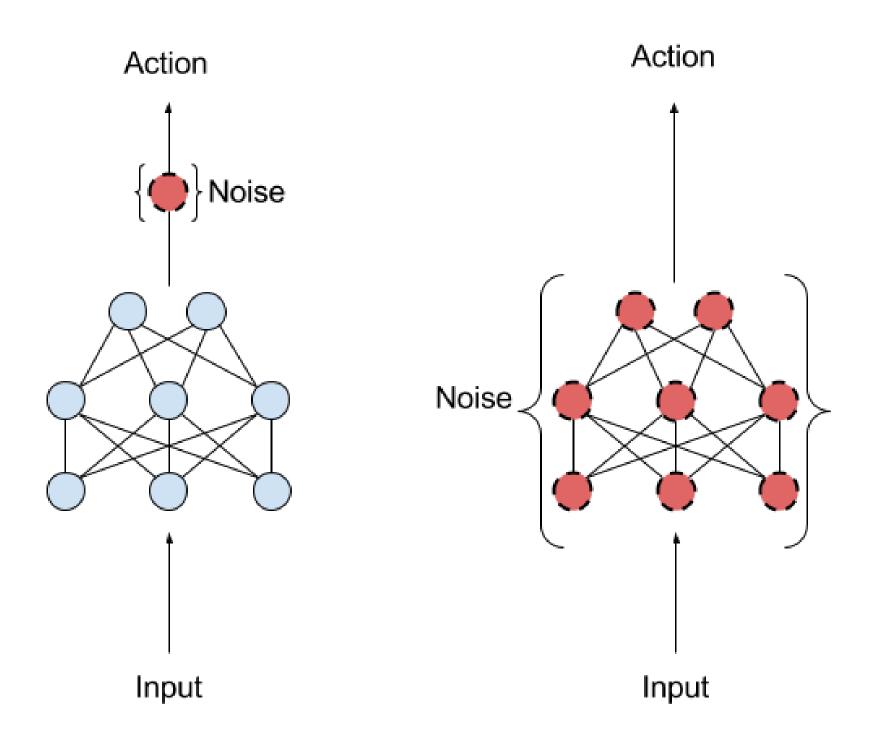
- In DDPG, an Ornstein-Uhlenbeck stochastic process is used to add noise to the continuous actions.
- It is defined by a **stochastic differential equation**, classically used to describe Brownian motion:

$$dx_t = heta(\mu - x_t)dt + \sigma dW_t \qquad ext{with} \qquad dW_t = \mathcal{N}(0, dt)$$

ullet The temporal mean of x_t is $\mu=0$, its amplitude is heta (exploration level), its speed is σ .



Parameter noise



Source: https://towardsdatascience.com/whats-new-in-deep-learning-research-knowledge-exploration-with-parameter-noise-98aef7ce84b2

- Another approach to ensure exploration is to add noise to the **parameters** θ of the actor at inference time.
- For the same input s_t , the output $\mu_{\theta}(s_t)$ will be different every time.
- The **NoisyNet** approach can be applied to any deep RL algorithm to enable a smart state-dependent exploration (e.g. Noisy DQN).

- Initialize actor network μ_{θ} and critic Q_{φ} , target networks $\mu_{\theta'}$ and $Q_{\varphi'}$, ERM ${\cal D}$ of maximal size N, random process ξ .
- ullet for $t\in [0,T_{\max}]$:
 - lacksquare Select the action $a_t = \mu_{ heta}(s_t) + \xi$ and store $(s_t, a_t, r_{t+1}, s_{t+1})$ in the ERM.
 - ullet For each transition (s_k,a_k,r_k,s_k') in a minibatch of K transitions randomly sampled from ${\cal D}$:
 - \circ Compute the target value using target networks $t_k = r_k + \gamma \, Q_{arphi'}(s_k', \mu_{ heta'}(s_k')).$
 - Update the critic by minimizing:

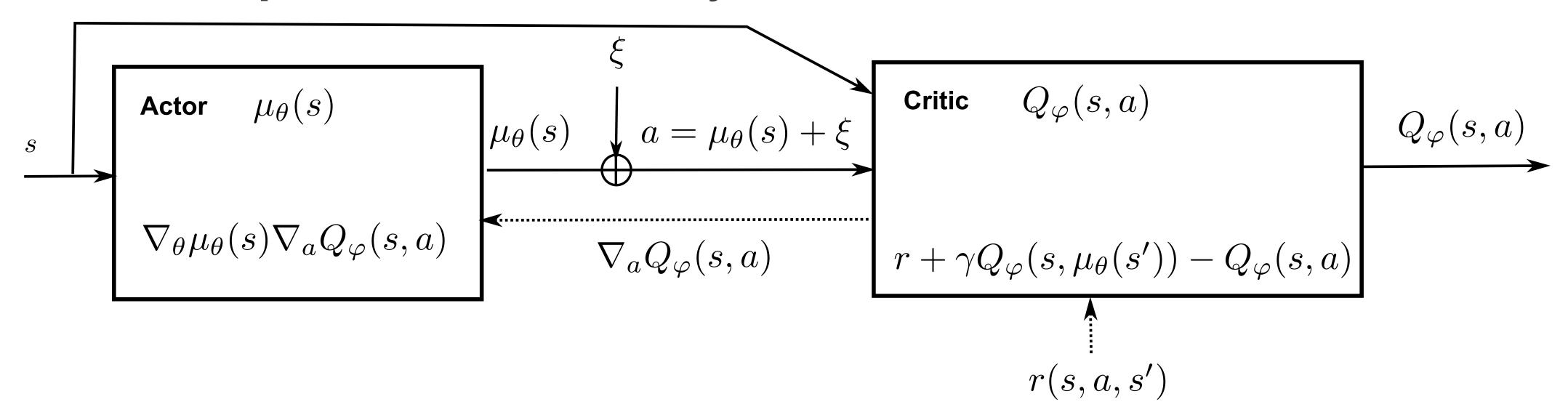
$$\mathcal{L}(arphi) = rac{1}{K} \sum_k (t_k - Q_arphi(s_k, a_k))^2$$

Update the actor by applying the deterministic policy gradient:

$$abla_{ heta} \mathcal{J}(heta) = rac{1}{K} \sum_k
abla_{ heta} \mu_{ heta}(s_k) imes
abla_a Q_{arphi}(s_k,a)|_{a=\mu_{ heta}(s_k)}$$

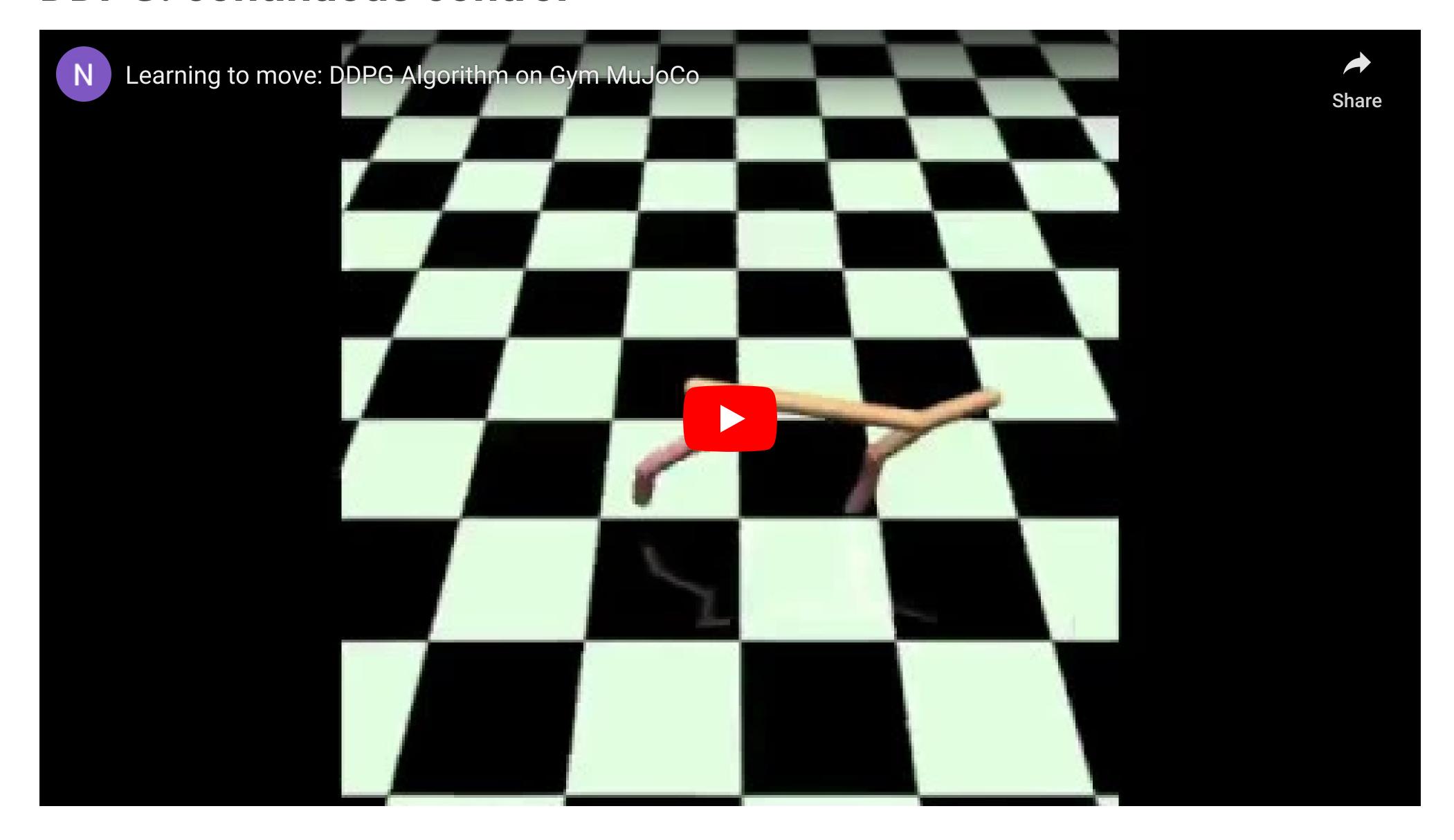
■ Update the target networks: $\theta' \leftarrow \tau\theta + (1-\tau)\,\theta'\;;\; \varphi' \leftarrow \tau\varphi + (1-\tau)\,\varphi'$

DDPG: Deep Deterministic Policy Gradient



- DDPG allows to learn continuous policies: there can be one tanh output neuron per joint in a robot.
- The learned policy is deterministic: this simplifies learning as we do not need to integrate over the action space after sampling.
- Exploratory noise (e.g. Ohrstein-Uhlenbeck) has to be added to the selected action during learning in order to ensure exploration.
- Allows to use an experience replay memory, reusing past samples (better sample complexity than A3C).

DDPG: continuous control



3 - DDPG: learning to drive in a day

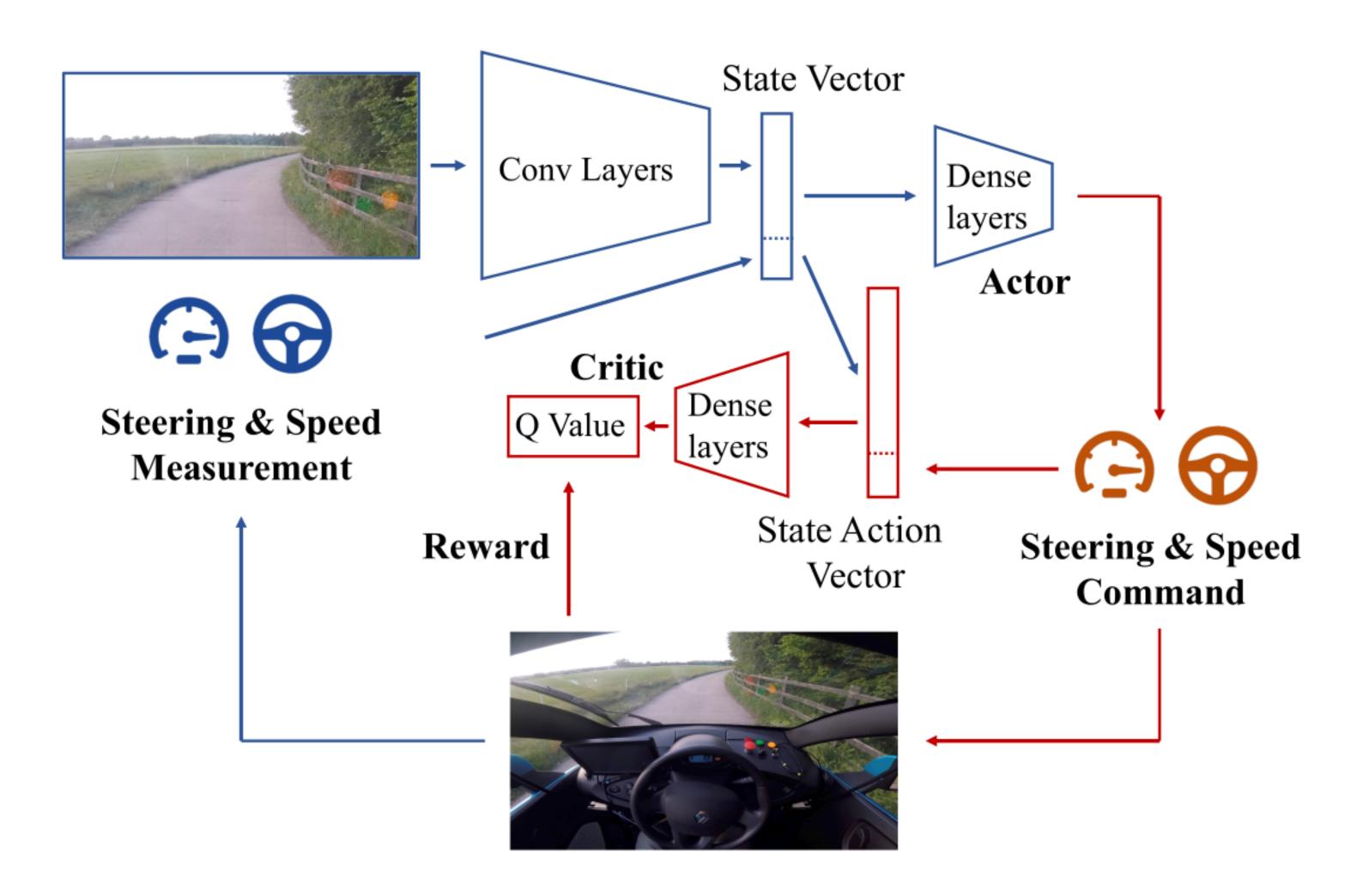
Learning to Drive in a Day

Alex Kendall Jeffrey Hawke David Janz Przemyslaw Mazur Daniele Reda John-Mark Allen Vinh-Dieu Lam Alex Bewley Amar Shah

DDPG: learning to drive in a day



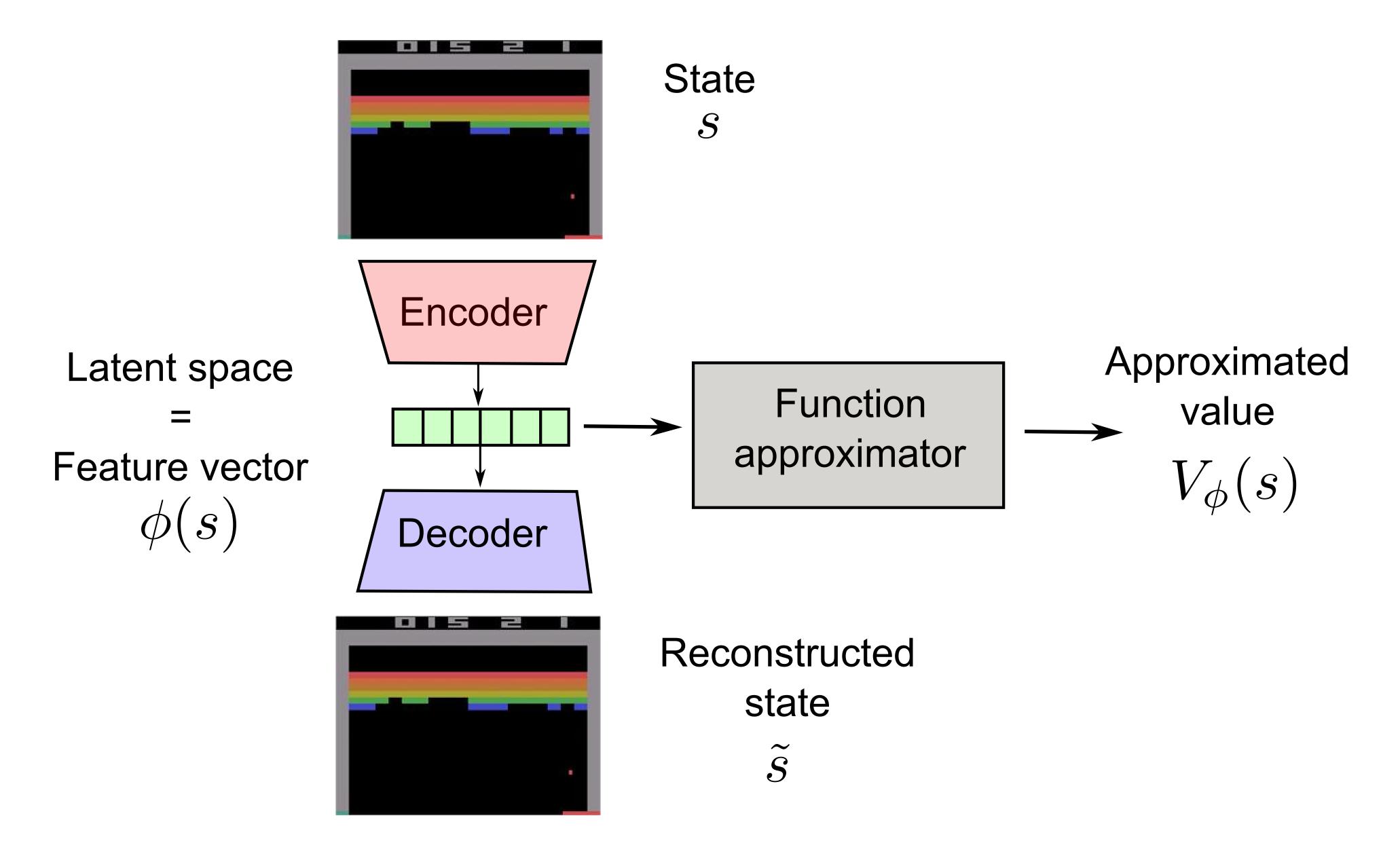
DDPG: learning to drive in a day



- The algorithm is DDPG with prioritized experience replay.
- Training is live, with an on-board NVIDIA Drive PX2 GPU.
- A simulated environment is first used to find the hyperparameters.

Autoencoders in deep RL

• A variational autoencoder (VAE) is optionally use to pretrain the convolutional layers on random episodes.



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DDPG: learning to drive in a day

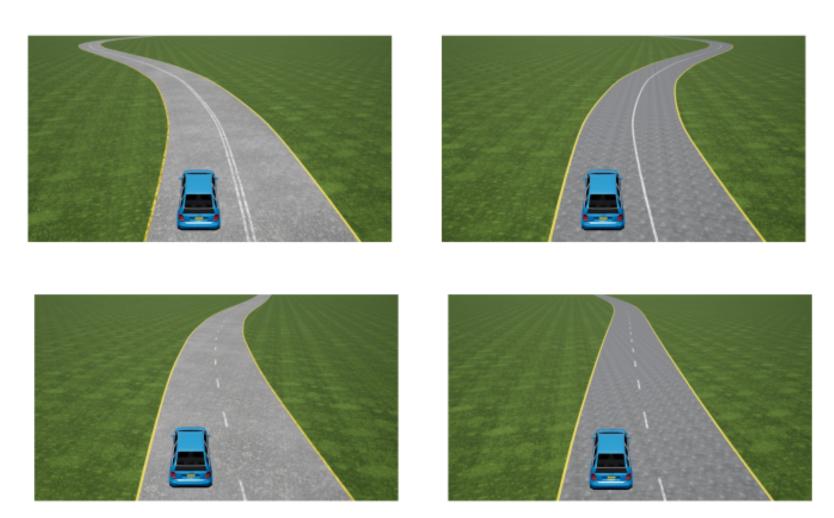


Fig. 3: Examples of different road environments randomly generated for each episode in our lane following simulator. We use procedural generation to randomly vary road texture, lane markings and road topology each episode. We train using a forward facing driver-view image as input.

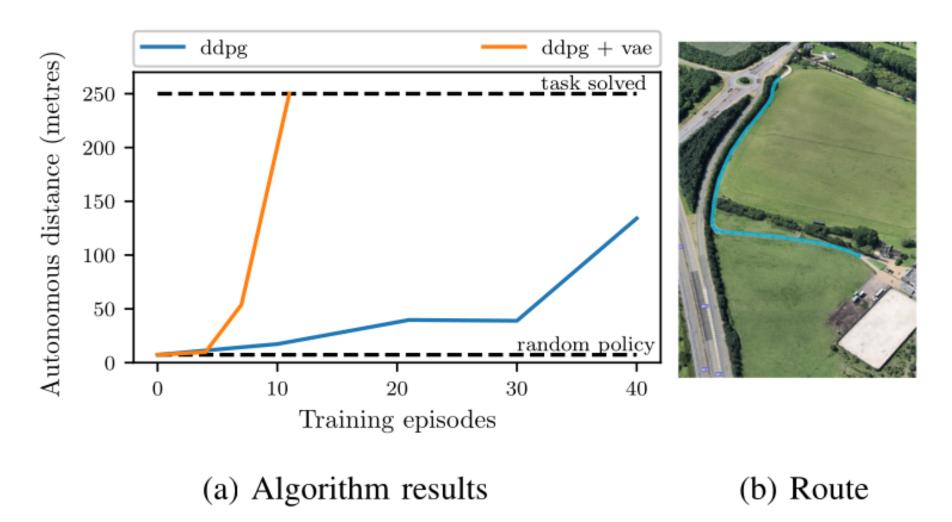


Fig. 4: Using a VAE with DDPG greatly improves data efficiency in training over DDPG from raw pixels, suggesting that state representation is an important consideration for applying reinforcement learning on real systems. The 250m driving route used for our experiments is shown on the right.

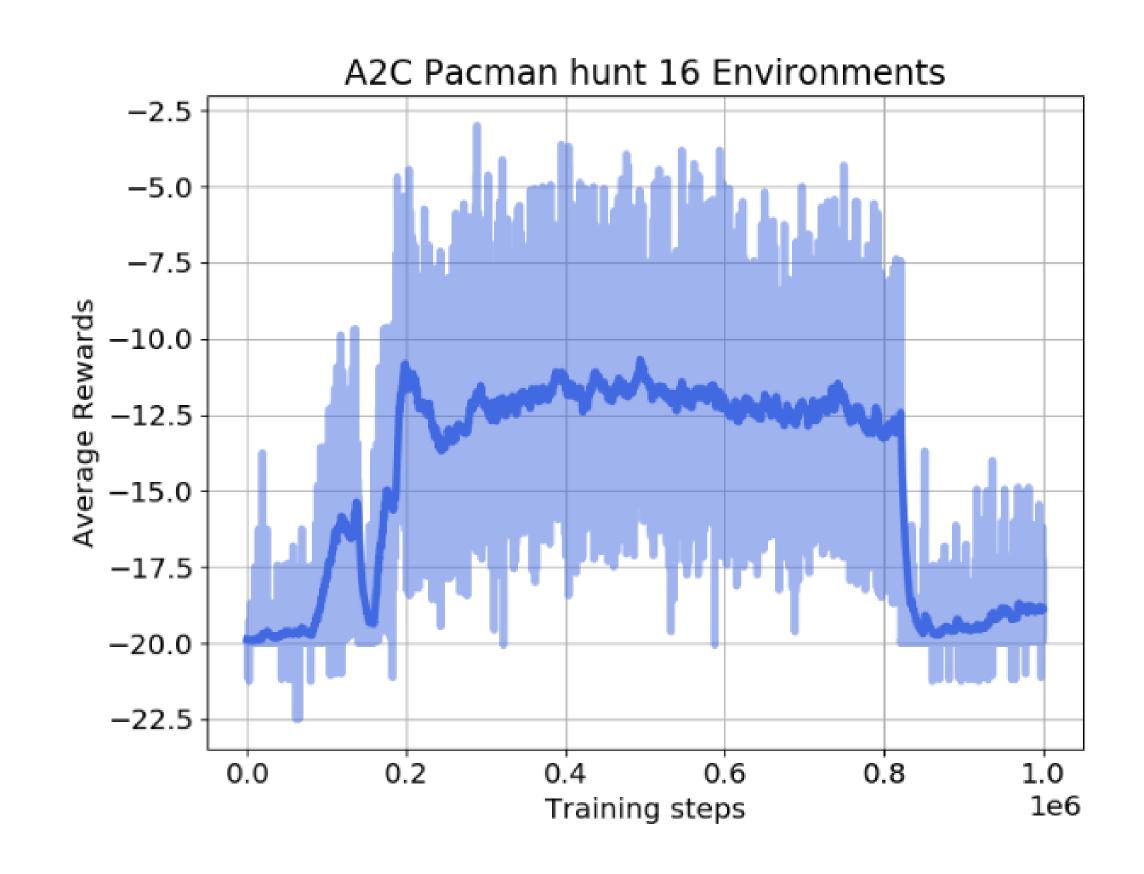
	Training			Test	
Model	Episodes	Distance	Time	Meters per Disengagement	# Disengagements
Random Policy	-	-	-	7.35	34
Zero Policy	-	-	-	22.7	11
Deep RL from Pixels	35	298.8 m	37 min	143.2	1
Deep RL from VAE	11	195.5 m	15 min	-	0

TABLE I: Deep reinforcement learning results on an autonomous vehicle over a 250m length of road. We report the best performance for each model. We observe the baseline RL agent can learn to lane follow from scratch, while the VAE variant is much more efficient, learning to successfully drive the route after only 11 training episodes.

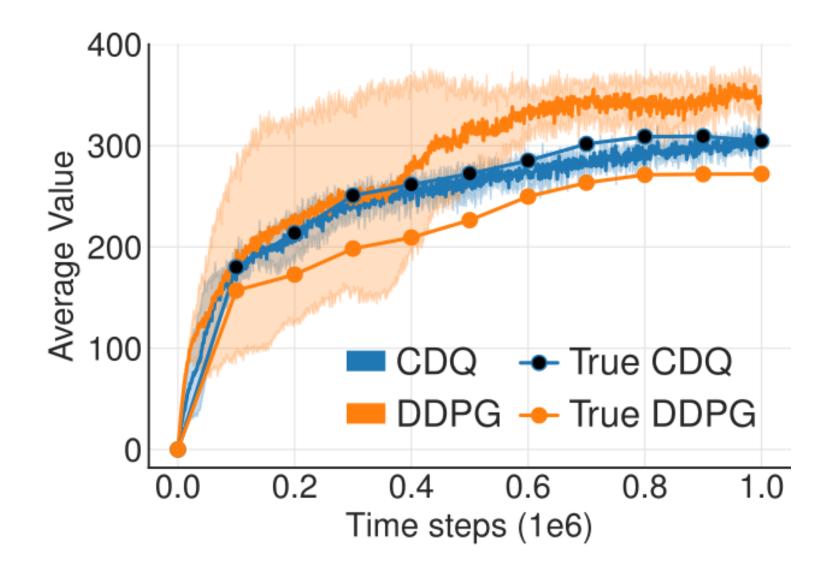
Addressing Function Approximation Error in Actor-Critic Methods

Scott Fujimoto¹ Herke van Hoof² David Meger¹

- DDPG suffers from several problems:
 - Unstable (catastrophic forgetting, policy collapse).
 - Brittleness (sensitivity to hyperparameters such as learning rates).
 - Overestimation of Q-values.
- Policy collapse happens when the bias of the critic is too high for the actor. Example with A2C:



- As any Q-learning-based method, DDPG **overestimates** Q-values.
- The Bellman target $t=r+\gamma \max_{a'} Q(s',a')$ uses a maximum over other values, so it is increasingly overestimated during learning.
- After a while, the overestimated Q-values disrupt training in the actor.



• Double Q-learning solves the problem by using the target network θ' to estimate Q-values, but the value network θ to select the greedy action in the next state:

$$\mathcal{L}(heta) = \mathbb{E}_{\mathcal{D}}[(r + \gamma \, Q_{ heta'}(s', ext{argmax}_{a'} Q_{ heta}(s', a')) - Q_{ heta}(s, a))^2]$$

- The idea is to use two different independent networks to reduce overestimation.
- This does not work well with DDPG, as the Bellman target $t=r+\gamma\,Q_{\varphi'}(s',\mu_{\theta'}(s'))$ uses a target actor network that is not very different from the trained deterministic actor.

- TD3 uses two critics φ_1 and φ_2 (and target critics):
 - the Q-value used to train the actor will be the lesser of two evils, i.e. the minimum Q-value:

$$t = r + \gamma \, \min(Q_{arphi_1'}(s', \mu_{ heta'}(s')), Q_{arphi_2'}(s', \mu_{ heta'}(s')))$$

- One of the critic will always be less over-estimating than the other.
 Better than nothing...
- Using twin critics is called clipped double learning.

The Lesser of Two Evils by Eric Perlin



Source: https://funnytimes.com/wp-content/uploads/2011/10/131986994517768.png

• Both critics learn in parallel using the same target:

$$\mathcal{L}(arphi_1) = \mathbb{E}[(t-Q_{arphi_1}(s,a))^2] \hspace{1cm} ; \hspace{1cm} \mathcal{L}(arphi_2) = \mathbb{E}[(t-Q_{arphi_2}(s,a))^2]$$

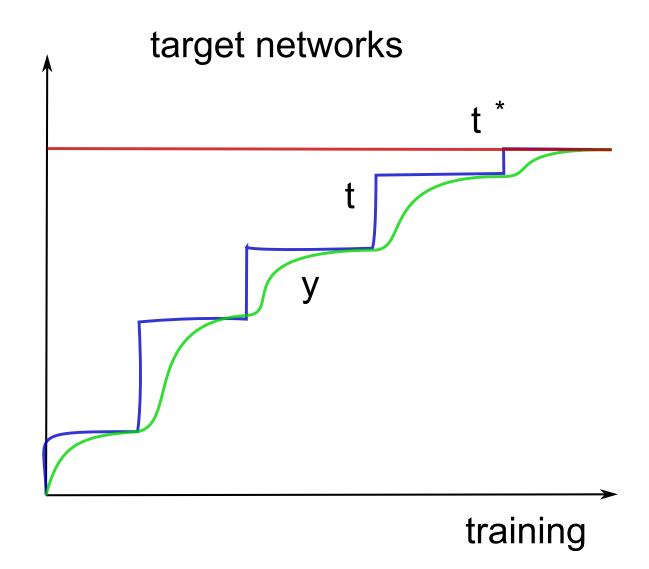
The actor is trained using the first critic only:

$$abla_{ heta} \mathcal{J}(heta) = \mathbb{E}[
abla_{ heta} \mu_{ heta}(s) imes
abla_{a} Q_{arphi_{1}}(s,a)|_{a=\mu_{ heta}(s)}]$$

=

• Another issue with actor-critic architecture in general is that the critic is always biased during training, what can impact the actor and ultimately collapse the policy:

$$egin{aligned}
abla_{ heta} \mathcal{J}(heta) &= \mathbb{E}[
abla_{ heta} \mu_{ heta}(s) imes
abla_{a} Q_{arphi_1}(s,a)|_{a=\mu_{ heta}(s)}] \ Q_{arphi_1}(s,a) &pprox Q^{\mu_{ heta}}(s,a) \end{aligned}$$



- The critic should learn much faster than the actor in order to provide unbiased gradients.
- Increasing the learning rate in the critic creates instability, reducing the learning rate in the actor slows down learning.
- ullet The solution proposed by TD3 is to **delay** the update of the actor, i.e. update it only every d minibatches:
 - Train the critics φ_1 and φ_2 on the minibatch.
 - every d steps:
 - \circ Train the actor θ on the minibatch.
- This leaves enough time to the critics to improve their prediction and provides less biased gradients to the actor.

- A last problem with deterministic policies is that they tend to always select the same actions $\mu_{\theta}(s)$ (overfitting).
- For exploration, some additive noise is added to the selected action:

$$a = \mu_{ heta}(s) + \xi$$

• But this is not true for the Bellman targets, which use the deterministic action:

$$t = r + \gamma \, Q_{arphi}(s', \mu_{ heta}(s'))$$

• TD3 proposes to also use additive noise in the Bellman targets:

$$t = r + \gamma \, Q_{arphi}(s', \mu_{ heta}(s') + \xi)$$

- If the additive noise is zero on average, the Bellman targets will be correct on average (unbiased) but will prevent overfitting of particular actions.
- The additive noise does not have to be an **Ornstein-Uhlenbeck** stochastic process, but could simply be a random variable:

$$\xi \sim \mathcal{N}(0,1)$$

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- Initialize actor $\mu_{ heta}$, critics Q_{arphi_1},Q_{arphi_2} , target networks $\mu_{ heta'},Q_{arphi'_1},Q_{arphi'_2}$, ERM \mathcal{D} , random processes ξ_1,ξ_2 .
- ullet for $t\in [0,T_{\max}]$:
 - lacksquare Select the action $a_t=\mu_{ heta}(s_t)+\xi_1$ and store $(s_t,a_t,r_{t+1},s_{t+1})$ in the ERM.
 - For each transition (s_k, a_k, r_k, s_k') in a minibatch sampled from \mathcal{D} :
 - \circ Compute the target $t_k=r_k+\gamma \, \min(Q_{arphi_1'}(s_k',\mu_{ heta'}(s_k')+\xi_2),Q_{arphi_2'}(s_k',\mu_{ heta'}(s_k')+\xi_2)).$
 - Update the critics by minimizing:

$$\mathcal{L}(arphi_1) = rac{1}{K} \sum_k (t_k - Q_{arphi_1}(s_k, a_k))^2 \qquad ; \qquad \mathcal{L}(arphi_2) = rac{1}{K} \sum_k (t_k - Q_{arphi_2}(s_k, a_k))^2$$

- every d steps:
 - \circ Update the actor by applying the DPG using Q_{arphi_1} :

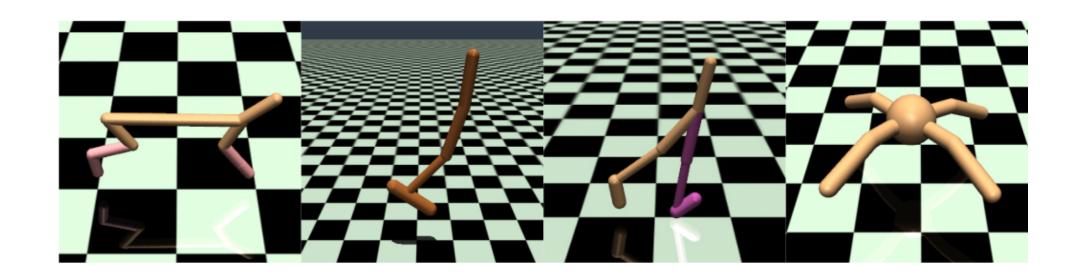
$$abla_{ heta} \mathcal{J}(heta) = rac{1}{K} \sum_k
abla_{ heta} \mu_{ heta}(s_k) imes
abla_a Q_{arphi_1}(s_k,a)|_{a=\mu_{ heta}(s_k)}$$

Update the target networks:

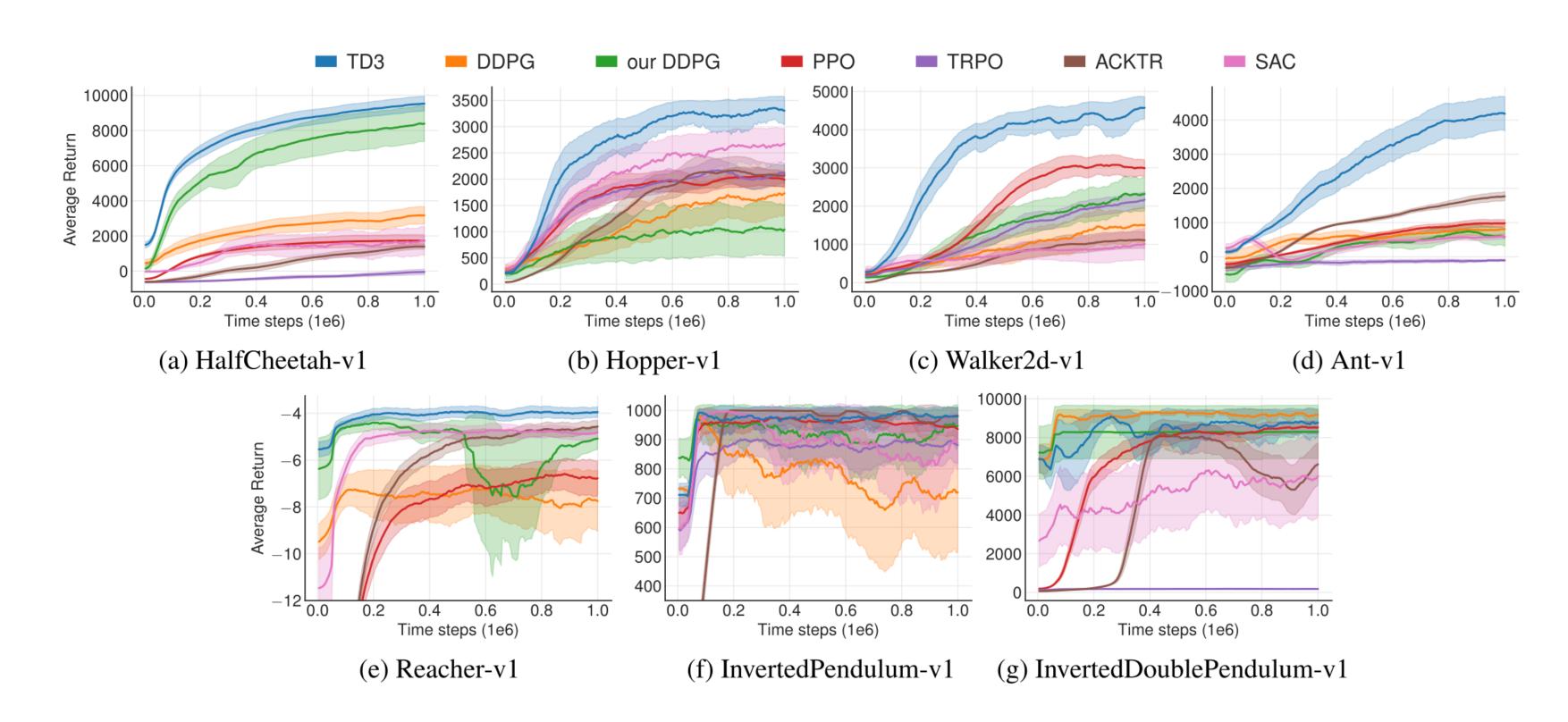
$$\theta' \leftarrow \tau\theta + (1-\tau)\theta'$$
; $\varphi_1' \leftarrow \tau\varphi_1 + (1-\tau)\varphi_1'$; $\varphi_2' \leftarrow \tau\varphi_2 + (1-\tau)\varphi_2'$

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- TD3 introduces three changes to DDPG:
 - twin critics.
 - delayed actor updates.
 - noisy Bellman targets.



• TD3 outperforms DDPG (but also PPO and SAC) on continuous control tasks.



5 - D4PG: Distributed Distributional DDPG

Published as a conference paper at ICLR 2018

DISTRIBUTED DISTRIBUTIONAL DETERMINISTIC POLICY GRADIENTS

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D4PG: Distributed Distributional DDPG

• Deterministic policy gradient as in DDPG:

$$abla_{ heta} \mathcal{J}(heta) = \mathbb{E}_{s \sim
ho_b} [
abla_{ heta} \mu_{ heta}(s) imes
abla_a \mathbb{E}[\mathcal{Z}_{arphi}(s,a)]|_{a=\mu_{ heta}(s)}]$$

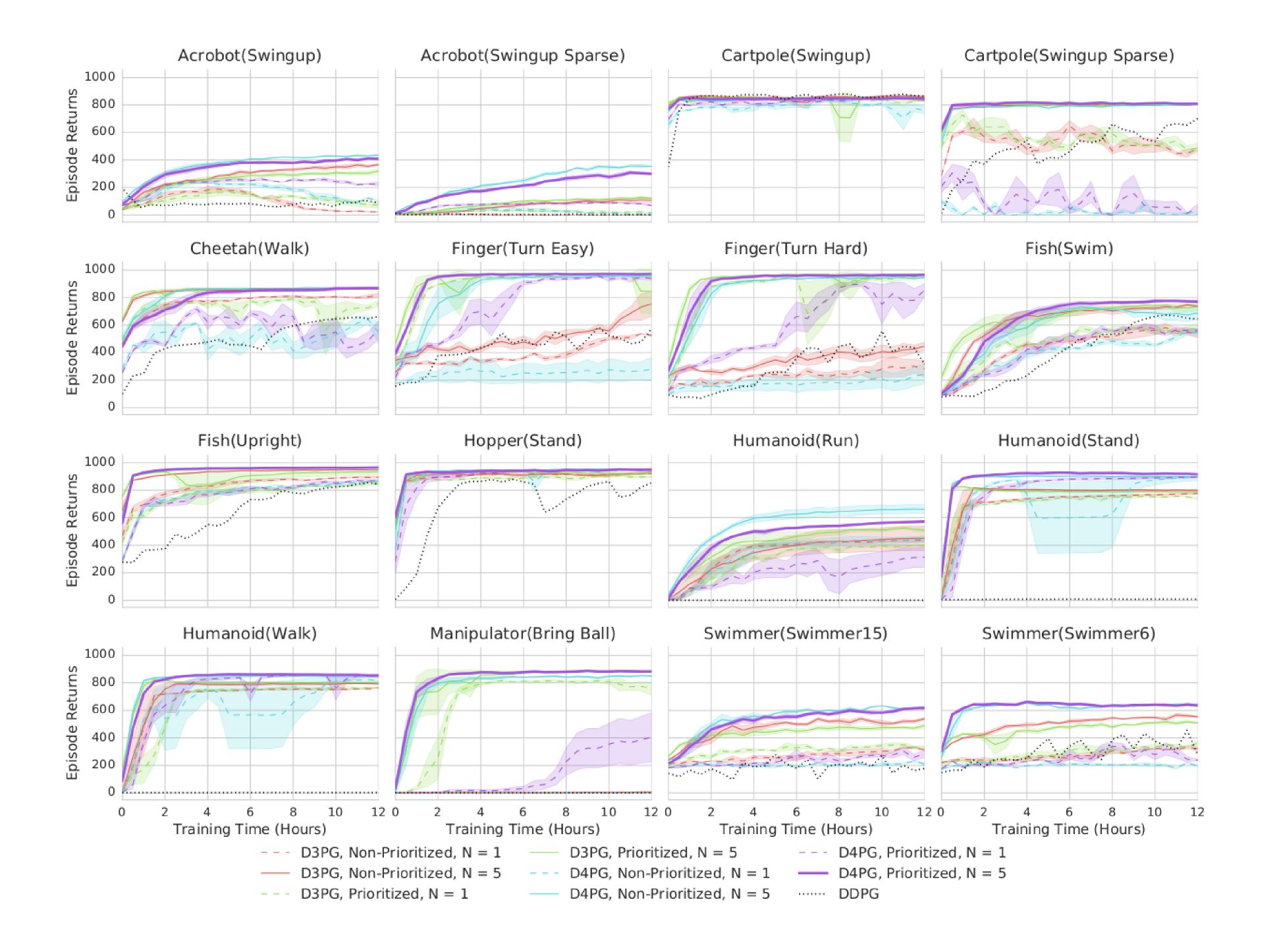
• **Distributional critic**: The critic does not predict single Q-values $Q_{\varphi}(s,a)$, but the distribution of returns $\mathcal{Z}_{\varphi}(s,a)$ (as in Categorical DQN):

$$\mathcal{L}(arphi) = \mathbb{E}_{s \in
ho_b}[ext{KL}(\mathcal{T}\,\mathcal{Z}_arphi(s,a)||\mathcal{Z}_arphi(s,a))]$$

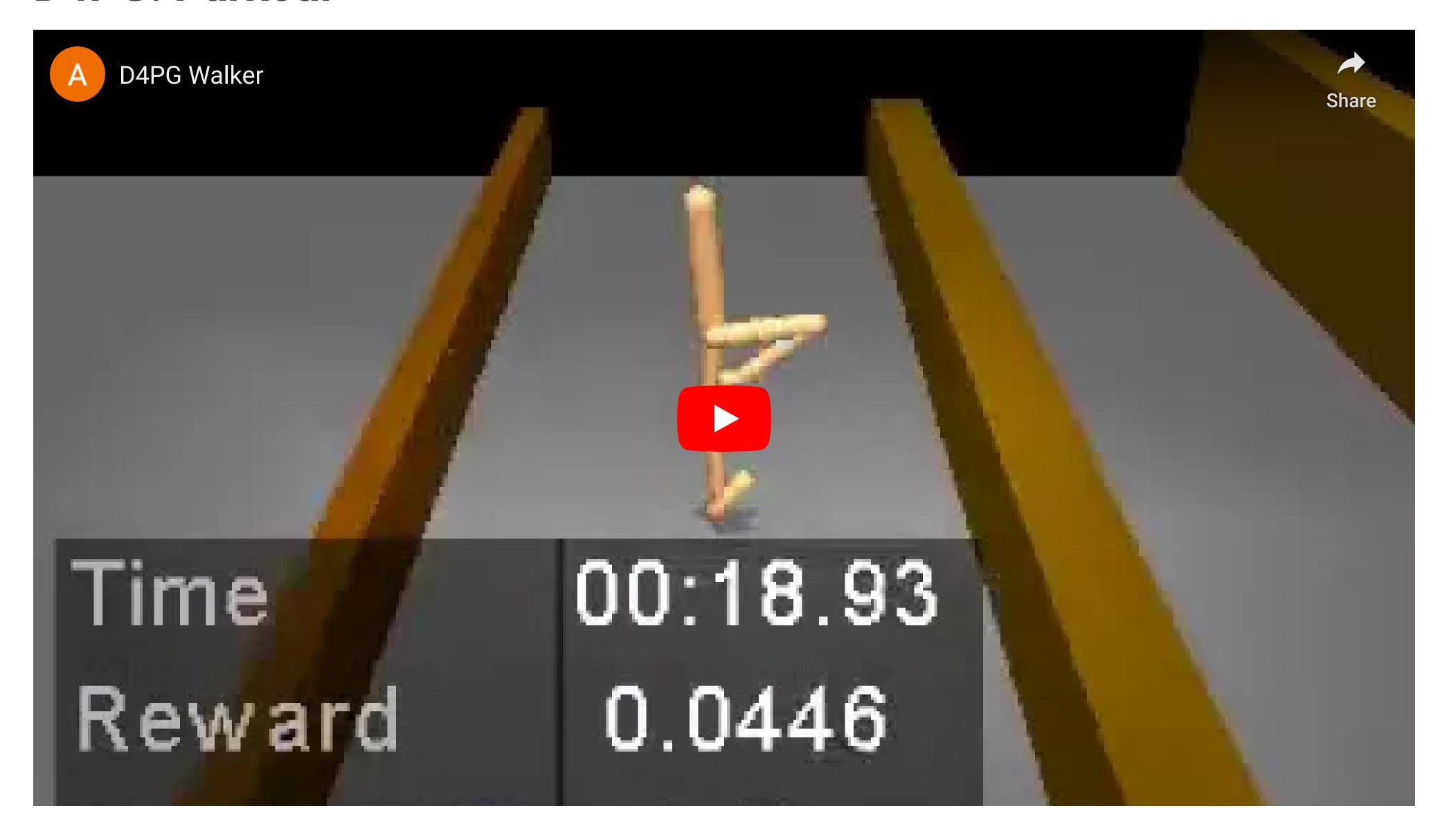
• **n-step** returns (as in A3C):

$$\mathcal{T}\mathcal{Z}_{arphi}(s_t,a_t) = \sum_{k=0}^{n-1} \gamma^k \, r_{t+k+1} + \gamma^n \, \mathcal{Z}_{arphi}(s_{t+n},\mu_{ heta}(s_{t+n}))$$

- ullet Distributed workers: D4PG uses K=32 or 64 copies of the actor to fill the ERM in parallel.
- Prioritized Experience Replay (PER): $P(k) = rac{(|\delta_k| + \epsilon)^{lpha}}{\sum_k (|\delta_k| + \epsilon)^{lpha}}$



D4PG: Parkour



Parkour networks

- For Parkour tasks, the states cover two different informations: the **terrain** (distance to obstacles, etc.) and the **proprioception** (joint positions of the agent).
- They enter the actor and critic networks at different locations.

